Multidimensional gravitational model with anisotropic pressure

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We consider the gravitational model with additional spatial dimensions and anisotropic pressure which is nonzero only in these dimensions. Cosmological solutions in this model include accelerated expansion of the Universe at late age of its evolution and dynamical compactification of extra dimensions. This model describes observational data for Type Ia supernovae on the level or better than the ΛCDM model. We analyze two equations of state resulting in different predictions for further evolution, but in both variants the acceleration epoch is finite.

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I. INTRODUCTION

The most important event of last 15 years in astrophysics is conclusion about accelerated expansion of our Universe at late stage of its evolution. This conclusion was based on observations of luminosity distances and redshifts for the Type Ia supernovae [1, 2], cosmic microwave background [3], large-scale galaxy clustering [4], and other evidence [5, 6].

To explain accelerated evolution of the Universe various mechanisms have been suggested, including the most popular cosmological model ΛCDM with a Λ term (dark energy) and cold dark matter (see reviews [5–8]). The ΛCDM model with 4% fraction of visible (baryonic) matter nowadays, 23% fraction of dark matter and 73% fraction of dark energy [3] describes Type Ia supernovae, data rather well and satisfies observational evidence, connected with rotational curves of galaxies, galaxy clusters and anisotropies of cosmic microwave background. However, the ΛCDM model (along with vague nature of dark matter and energy) has some problems with fine tuning of the observed value of Λ, which is many orders of magnitude smaller than expected vacuum energy density, and with different time dependence of dark energy Ω_Λ and material Ω_m fractions (we have $\Omega_\Lambda \simeq \Omega_m$ nowadays).

Therefore a large number of alternative cosmological models have been proposed. They include matter with nontrivial equations of state, for example, Chaplygin gas and modified equations of state [9–11]; scalar fields with a potential [12–14]; modified gravity with $f(R)$ Lagrangian [15, 16]; theories with extra dimensions, in particular, brane models [17] and many others [5–8].

In this paper we explore the cosmological model with anisotropic pressure and nontrivial equation of state in $1+3+d$ dimensions, suggested by Pahwa, Choudhury and Seshadri in Ref. [9]. The authors omitted the important case $d = 1$, we include it into consideration. We also analyse how to modify the equation of state and to avoid "the end of the world" (the finite-time future singularity) which is inevitable in the model [9].

In this model the $1 + 3 + d$ dimensional spacetime is symmetric and isotropic in two subspaces: in 3 usual spatial dimensions and in d extra dimensions. It has the following metric with two Robertson–Walker terms:

$$\begin{aligned} ds^2 \;&=\; -dt^2 + a^2(t)\left(\frac{dr^2}{1-k_1r^2} + r^2 d\Omega\right) \\ &+\; b^2(t)\left(\frac{dR^2}{1-k_2R^2} + R^2 d\Omega_{d-1}\right). \end{aligned} \tag{1.1}$$

Here the signature is $(-,+,\ldots,+)$, the speed of light $c = 1$, $a(t)$ and k_1 is are the scale factor and curvature sign in usual dimensions, $b(t)$ and k_2 are corresponding values for extra dimensions. It is supposed that the scale factor $a(t)$ grows while $b(t)$ diminishes, in other words, some form of dynamical compactification takes place.

Authors of Ref. [9] develop the approach of Ref. [18] and suppose that the spacetime (1.1) is filled with a uniform density matter with anisotropic pressure and the following energy-momentum tensor:

$$T^\mu_\nu = \mathrm{diag}\,(-\rho, P_a, P_a, P_a, P_b, \ldots, P_b). \tag{1.2}$$

Here ρ is the energy density and P_a (P_b) is the pressure in normal (extra) dimensions. So in normal dimensions pressure is different from that in additional dimensions, while being isotropic within each subspace.

In Ref. [9] matter is supposed to behave like pressureless dust ($P_a = 0$) in usual dimensions, while in extra dimensions it has appreciable pressure P_b depending on density ρ by a power law

$$P_b = W\rho^{1-\gamma} \tag{1.3}$$

with a negative constant W. In this model matter (1.2) with anisotropic pressure plays a role of dark energy and source of accelerated expansion. So the following Einstein equation without usual Λ term is considered:

$$G^\mu_\nu = 8\pi G T^\mu_\nu. \tag{1.4}$$

To describe accelerated expansion of the Universe authors of Ref. [9] used cosmological solutions of their model with $d \geq 2$ additional dimensions. In Sect. II of

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this paper we show, that the case $d = 1$ is also applicable to describe acceleration. In Sect. III we apply this model for all $d \geq 1$ to describing observational data for Type Ia supernovae and determine optimal model parameters. In Sect. IV we modify the model [9] to solve the above mentioned problem of "the end of the world".

II. COSMOLOGICAL SOLUTIONS

For the considered metric (1.1) in the case $k_2 = 0$ the Einstein tensor components G^μ_ν ($\mu, \nu = 0, 1, \ldots, d+3$, $1 \leq i \leq 3 < I$) are [9]:

$$\begin{aligned}
G^0_0 &= -3d\frac{\dot a}{a}\frac{\dot b}{b} - 3\frac{\dot a^2}{a^2} - \frac{d(d-1)}{2}\frac{\dot b^2}{b^2} - 3\frac{k_1}{a^2},\\
G^i_i &= -2\frac{\ddot a}{a} - d\frac{\ddot b}{b} - 2d\frac{\dot a}{a}\frac{\dot b}{b} - \frac{\dot a^2}{a^2} - \frac{d(d-1)}{2}\frac{\dot b^2}{b^2} - \frac{k_1}{a^2},\\
G^I_I &= (1-d)\left[\frac{\ddot b}{b} + 3\frac{\dot a}{a}\frac{\dot b}{b} + \left(\frac{d}{2} - 1\right)\frac{\dot b^2}{b^2}\right] - 3\frac{\ddot a a + \dot a^2 + k_1}{a^2}.
\end{aligned}$$

If we substitute these expressions into Eq. (1.4) and add the continuity condition $T^\mu_{\nu;\mu} = 0$ we obtain the system of cosmological equations. This system has the form

$$\begin{aligned}
\frac{\dot a^2}{a^2} + \frac{\dot a}{a}\frac{\dot b}{b} + \frac{k_1}{a^2} &= \frac{8\pi G}{3}\rho, && (2.1)\\
2\frac{\ddot a}{a} + \frac{\ddot b}{b} + 2\frac{\dot a}{a}\frac{\dot b}{b} + \frac{\dot a^2}{a^2} + \frac{k_1}{a^2} &= 0, && (2.2)\\
-\frac{\ddot a a + \dot a^2 + k_1}{a^2} &= \frac{8\pi G}{3}P_b, && (2.3)\\
\frac{d}{dt}(\rho a^3 b) + P_b a^3 \frac{d}{dt} b &= 0. && (2.4)
\end{aligned}$$

in the case with $d = 1$ extra spatial dimension, that did not considered in Ref. [9]. Here pressure P_a in "usual" dimension equals zero, as mentioned above. Eq. (2.4) is the continuity condition for $d = 1$ and $P_a = 0$.

Using the Hubble constant $H_0 \simeq 2.28 \cdot 10^{-18}$ c^{-1} [3] and the critical density

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{2.5}$$

at the present time, we make the following substitutions

$$\tau = H_0 t, \quad \bar\rho = \frac{\rho}{\rho_c}, \quad \bar p_b = \frac{P_b}{\rho_c}, \quad A = \log\frac{a}{a_0}, \quad B = \log\frac{b}{b_0} \tag{2.6}$$

and introduce dimensionless time τ, density $\bar\rho$, pressure $\bar p_b$ and logarithms A, B of the scale factors (here a_0, b_0 are present time values of a and b).

We denote derivative with respect to τ as primes and rewrite the system (2.1) – (2.4) as follows:

$$\begin{aligned}
A'^2 + A'B' - \Omega_k e^{-2A} &= \bar\rho, && (2.7)\\
2A'' + 3A'^2 + B'' + B'^2 + 2A'B' &= \Omega_k e^{-2A}, && (2.8)\\
A'' + 2A'^2 - \Omega_k e^{-2A} &= -\bar p_b, && (2.9)\\
\bar\rho' + 3\bar\rho A' + (\bar\rho + \bar p_b)\, B' &= 0. && (2.10)
\end{aligned}$$

Here

$$\Omega_k = -k_1 (a_0 H_0)^{-2}. \tag{2.11}$$

If we express

$$B' = (\bar\rho + \Omega_k e^{-2A})/A' - A' \tag{2.12}$$

from Eq. (2.7) and substitute it into three equations (2.8) – (2.10), one should note that Eq. (2.8) may be reduced to Eq. (2.9). So in the planar case

$$k_1 = k_2 = 0, \qquad \Omega_k = 0$$

we have the system of two independent equations

$$\begin{aligned}
A'' &= -2A'^2 - \bar p_b, && (2.13)\\
\bar\rho' &= -3\bar\rho A' + (\bar\rho + \bar p_b)(A' - \bar\rho/A'). && (2.14)
\end{aligned}$$

If we fix an equation of state for pressure $\bar p_b$, for example, the above mentioned power law (1.3)

$$\bar p_b = w\bar\rho^{1-\gamma}, \tag{2.15}$$

we may consider the equations (2.13), (2.14) as a closed system of first order differential equations with respect to 2 unknown functions $A'(\tau)$ and $\bar\rho(\tau)$. The dependence (2.15) is used in Ref. [9], where parameters w and γ are chosen in accordance with observations.

The Cauchy problem for the system (2.13), (2.14) requires two initial conditions. We refer them to the present epoch (here and below it corresponds to the value $\tau = 1$) in the following form:

$$A'\big|_{\tau=1} = 1, \qquad \bar\rho\big|_{\tau=1} = \Omega_0. \tag{2.16}$$

The first condition results from definition of the Hubble constant

$$H_0 = \frac{\dot a}{a}\Big|_{t=t_0} = H_0 A'\big|_{\tau=1}.$$

In the second condition (2.16) we suppose that the energy density $\rho = \bar\rho \cdot \rho_c$ at the present time has the fraction Ω_0 in the critical density (2.5). In Ref. [9] this fraction equals matter density fraction in the ΛCDM model [6]:

$$\Omega_0 = \Omega_m = 0.27. \tag{2.17}$$

Note that in Ref. [9] the second condition (2.16) was used in the form $\bar\rho\big|_{\tau=1} = 1$, but the value Ω_0 (2.17) was taken as the factor in the r.h.s. of Eq. (1.4). From our point of view, that approach introduces useless vagueness in physical sense of the value ρ. In our approach ρ in conditions (2.16) is density of all gravitating matter (visible and dark) with described above anisotropic pressure.

Remind that we have no dark energy or Λ term in Eq. (1.4) in the model [9]. Anisotropic pressure in additional dimensions plays here the role of dark energy as a source of acceleration. The contribution of this source is the term $\Omega_B = -B'\big|_{\tau=1}$ in the equality

$$\Omega_m + \Omega_B + \Omega_k = 1, \tag{2.18}$$

that results from equation (2.7), if we fix it at the present time $\tau = 1$.

To obtain cosmological solutions for $d = 1$, $k_1 = 0$ in this model we are to solve numerically the Cauchy problem for the system (2.13), (2.14) with initial conditions (2.16) moving into the past for $\tau < 1$ and into the future for $\tau > 1$. Then we integrate functions $A'(\tau)$ and $B'(\tau)$ (2.12) keeping in mind Eqs. (2.6) and calculate dependence of the scale factors $a = a_0 e^A$, $b = b_0 e^B$ and density $\bar\rho$ on dimensionless time τ.

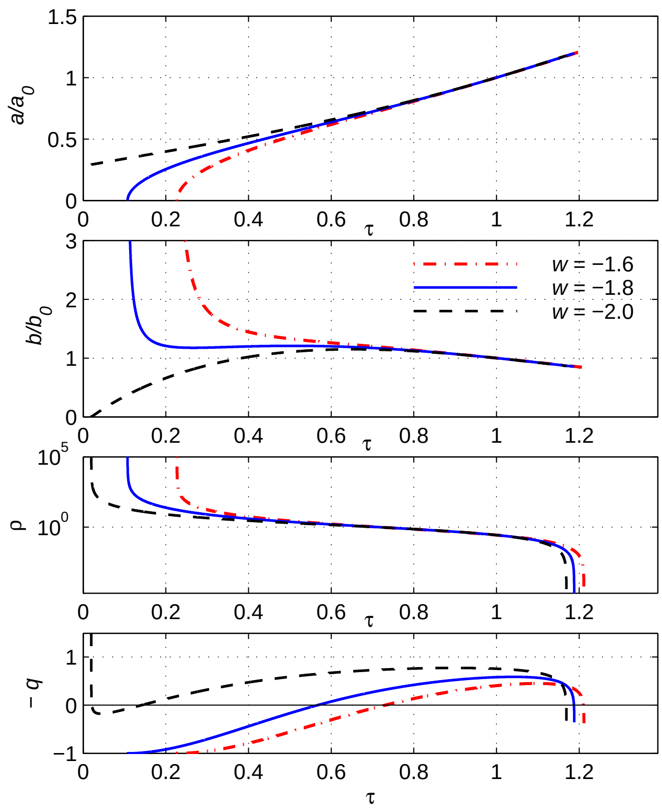

FIG. 1: Scale factors a,b, density $\bar\rho$ and acceleration parameter $-q$ depending on dimensionless time τ for $\Omega_0 = 0.27$, $\gamma = 0.9$ and specified values of w

The results of calculation for scale factors $a(\tau)$, $b(\tau)$, density $\bar\rho(\tau)$ and the acceleration parameter

$$-q = \frac{\ddot a a}{\dot a^2} = \frac{A'' + A'^2}{A'^2} \tag{2.19}$$

(q is the deceleration parameter) are presented in Fig. 1. Here $k_1 = 0$, $\Omega_0 = 0.27$, $\gamma = 0.9$ and 3 scenarios for $w = -1.6$ (dash-dotted line), $w = -1.8$ (solid lines) and $w = -2$ (dashed lines) are shown.

This evolution begins from infinite value of density $\bar\rho$ at some initial moment τ_0. We can see here two different variants for this beginning. For solutions with $w = -1.6$ and $w = -1.8$ (we denote them as "regular" solutions) the scale factor a expands from $a = 0$ like $a \sim \sqrt{\tau - \tau_0}$ at the initial stage whereas the scale factor b diminishes from initial infinite value up to values $b \simeq b_0$ during some percent of total lifetime of this universe. This behavior of $b(\tau)$ looks like some variant of dynamical compactification, because the parameter b_0 is arbitrary one in this model, we may put b_0 to be sufficiently small.

Another type of evolution ("singular" solutions) is represented with dashed lines in Fig. 1 for $w = -2$. For singular solutions infinite value of density $\bar\rho$ at $\tau = \tau_0$ corresponds to nonzero value of the scale factor a and $b = 0$. Obviously, these solutions are nonphysical and should be excluded.

All regular and singular solutions in Fig. 1 describe accelerated expansion (for the factor a) at late stage of evolution. Beginning of this stage may be seen in the graph of the acceleration parameter $-q(\tau)$. Acceleration rate depends on the parameters w, γ, Ω_0 and the curvature fraction (2.11) $\Omega_k = -k_1(a_0H_0)^{-2}$ depending on the sign k_1. If $\Omega_k \neq 0$ ($k_1 = \pm1$), one should use the system (2.9) – (2.12) instead of Eqs. (2.13), (2.14). In this case we integrate numerically the function $A'(\tau)$ simultaneously with solving the Cauchy problem for the system (2.9) – (2.12). We add here the natural initial condition $A\big|_{\tau=1} = 0$ to conditions (2.16).

For all reasonable values of four free parameters w, γ, Ω_0, Ω_k the stage of accelerated expansion appears to be finite, because density $\bar\rho$ inevitably vanishes in this model. In Fig. 1 this effect may be seen in the graphs $\bar\rho(\tau)$ with logarithmic scale in Y-direction. We denote the moment of zero density by τ_*: $\bar\rho(\tau_*) = 0$. For $\tau > \tau_*$ density $\bar\rho$ becomes negative and nonphysical, all energy conditions (in particular, the weak energy condition) are violated.

This finite-time future singularity may be classified as the Type IV singularity in accordance with the scheme from Refs. [7, 16]. For this singularity $a(\tau_*)$ is nonzero, $\bar\rho(\tau_*)$ equals zero, the effective density and pressure

$$\rho_{eff} = \frac{3H^2}{8\pi G} = \rho_c A'^2, \quad p_{eff} = -\frac{2\dot H + 3H^2}{8\pi G} = \frac{2q-1}{3}\rho_{eff}$$

remain nonzero, but higher derivatives of H diverge at $\tau \to \tau_*$.

Note that the main features of the considered cosmological solutions, in particular, the future singularity, finite lifetime $\tau_0 \le \tau \le \tau_*$ and negative density for $\tau > \tau_*$ take place not only for $d = 1$, but also for higher dimensions $d \ge 2$. In the case of $d \ge 2$ additional dimensions after substituting the components G^μ_ν into Einstein equation (1.4) and substitutions (2.6) in these equations and Eq. (2.4) we have in the flat case $k_1 = k_2 = 0$ the following system [9], generalizing Eqs. (2.12) – (2.14):

$$\begin{aligned} A'' &= \frac{d(d-1)\,B'\big(\frac12 B' - A'\big) - 3(d+1)\,A'^2 - 3d\bar p_b}{d+2}, \\ \bar\rho' &= -3\bar\rho A' - d(\bar\rho + \bar p_b)\,B', \\ B' &= \frac{\sqrt{3\big[(d+2)\,A'^2 + 2(d-1)\,\bar\rho\big]/d} - 3A'}{d-1}. \end{aligned} \tag{2.20}$$

Solutions of the system (2.20) for $d \geq 2$ were obtained in Ref. [9], but some features of them were not considered in that paper. For example, singular solutions with nonzero value $a(\tau_0)$ (where $\bar\rho$ is infinite at the initial moment τ_0) also take place for $d \geq 2$, if the value w is less than the critical value $w_{cr}(\gamma, \Omega_0)$. In Fig. 2 boundaries $w = w_{cr}$ separating domains of regular and singular solutions on the γ, w plane are presented for different d and Ω_0. Singular solutions are described by the inequality $w < w_{cr}(\gamma, \Omega_0)$ and lie below corresponding lines in Fig. 2.

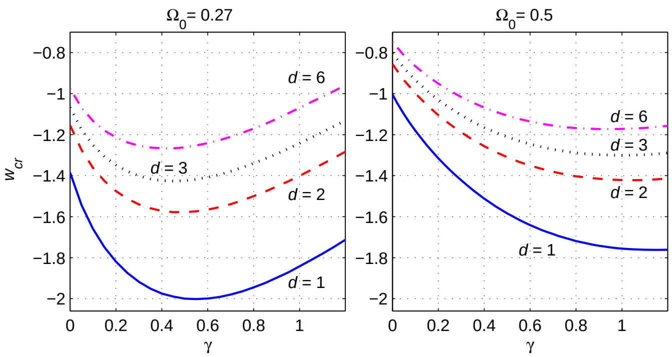

FIG. 2: Boundaries $w = w_{cr}$ between domains of regular (above) and singular solutions (below a curve) for indicated values Ω_0 and d

Another important property of these cosmological solutions is their finite-time future singularity, in other words, inevitability of "the end of the world" because of vanishing density at $\tau = \tau_*$ for all d (see Fig. 4 below). The authors of Ref. [9] did not pay attention to this phenomenon, essential for their model. It is connected with the chosen equation of state (2.15) for pressure $\bar p_b$ in extra dimensions. This drawback will be eliminated with modifying the model [9] in Sect. IV after application this model to describing observational data for Type Ia supernovae in the next section.

III. APPLICATION TO SUPERNOVAE OBSERVATIONS

To apply the model to describing the observational data it is convenient, following the authors of [9], to use Internet table [19] for Type Ia supernovae in distant galaxies. At the present moment this updated table contains redshifts $z = z_i$, distance moduli μ_i and errors σ_i of μ_i for $N = 580$ supernovae.

Redshift

$$z = \frac{a_0}{a(t)} - 1 = e^{-A(\tau)} - 1 \tag{3.1}$$

is associated with the value of a at the time t of a supernova light emission. The distance modulus μ is the logarithmic function

$$\mu = 5 \log \frac{D_L}{10 \text{ pc}},$$

of the luminosity distance [6, 9]:

$$D_L = (1+z) \int\limits_z^0 \frac{d\tilde z}{H(\tilde z)} = \frac{a_0^2}{H_0 a(\tau)} \int\limits_\tau^1 \frac{d\tilde\tau}{a(\tilde\tau)}. \tag{3.2}$$

To describe the data [19] of Type Ia supernovae, for given values d, w, γ, Ω_0 of this model we consider evolution of the scale factor $a(\tau)$ and dependence of the numerical integral (3.2) D_L and μ on τ. For each value of redshift z_i in the table [19] we calculate the corresponding $\tau = \tau_i$ with using Eq. (3.1) and linear approximation and the theoretical value $\mu_{th} = \mu(\tau_i)$ for τ_i from Eq. (3.2). The measure of differences between these theoretical values $\mu_{th} = \mu_{th}(d, w, \gamma, \Omega_0, \Omega_k, z_i)$ and the measured values μ_i is [9]:

$$\chi^2(d, w, \gamma, \Omega_0, \Omega_k) = \sum_{i=1}^N \frac{\big[\mu_i - \mu_{th}(d, \dots, z_i)\big]^2}{\sigma_i^2}. \tag{3.3}$$

The authors of Ref. [9] calculated optimal parameters w and γ, minimizing the function (3.3) for the flat model ($k_1 = 0$) with fixed $\Omega_0 = 0.27$ (2.17) and $d \geq 2$. In this approach for each $d \geq 2$ they minimized the function $\chi^2(w, \gamma)$ of two variables.

We generalize their approach to the case $d = 1$ additional dimension. At the first step we fix $k_1 = 0$, $\Omega_0 = 0.27$ in according with Ref. [9] and obtain the picture of level lines for the function $\chi^2(w, \gamma)$, presented in Fig. 3 for $d = 1$ and $d = 2$.

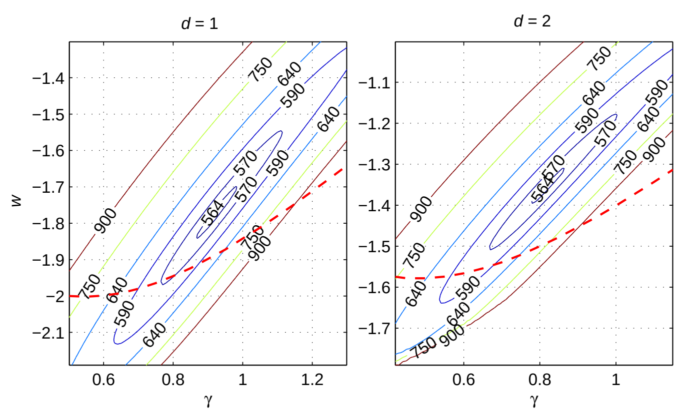

FIG. 3: Level lines of $\chi^2(w, \gamma)$ for $k_1 = 0$, $\Omega_0 = 0.27$. The dashed line is the boundary of singular solutions

Here the dashed line is taken from Fig. 2 and separates regular and singular solutions. We see that for $d = 1$ and $d = 2$ the minimum of χ^2 lies above this line, that is in the domain of regular solutions. The same picture also takes place for $d \geq 3$.

For each $d \geq 1$ we calculated minimums for the function of two variables $\chi^2(w, \gamma)$ and coordinates w, γ of this minimum. They are represented in Table I.

We compare these minimal values with the value of the function (3.3) for the flat ΛCDM model with the

TABLE I: Minimum of χ^2 and optimal values w, γ for fixed $k_1 = 0$, $\Omega_0 = 0.27$ and various d

d	1	2	3	6	10	ΛCDM
$\min\chi^2$	563.15	563.41	563.52	563.65	563.7	563.29
w	-1.77	-1.355	-1.212	-1.067	-1.01	-
γ	0.925	0.819	0.772	0.715	0.686	-

same parameters $k_1 = 0$, $\Omega_m = \Omega_0 = 0.27$ (therefore, $\Omega_\Lambda = 0.73$) and the same supernova data [19]. We see that the predictions are rather close, and for $d = 1$ the model [9] fits the data better than the flat ΛCDM model.

At the next step for more precise estimation of optimal model parameters we consider variations of fractions $\Omega_0 = \Omega_m$ and Ω_k for matter density and curvature respectively. One should take into account these degrees of freedom in both models: the model [9] and ΛCDM. In the model [9] for each $d \geq 1$ we minimize the function (3.3) of four variables: $\chi^2(w, \gamma, \Omega_0, \Omega_k)$. We also compare this results with the same value of the ΛCDM model (where χ^2 depends on Ω_m and Ω_k) and keep in mind the constraints on these parameters due to cosmic microwave background anisotropy, galaxy clustering and other factors [3]:

$$\Omega_m = 0.2743 \pm 0.0072, \quad -0.0133 < \Omega_k < 0.0084. \quad (3.4)$$

Numerical search of this minimum includes a starting point (for example, the values from Table I), analysis of gradients or increments for χ^2 and the constraints (3.4). The results of calculation with optimal values of the model parameters are presented in Table II.

TABLE II: Minimum of χ^2 and optimal values w, γ, Ω_0, Ω_k

d	1	2	3	6	10	ΛCDM
$\min\chi^2$	563.136	563.39	563.506	563.634	563.69	563.058
w	-1.740	-1.323	-1.2032	-1.061	-1.003	-
γ	0.926	0.821	0.7739	0.7174	0.696	-
Ω_0	0.2815	0.279	0.274	0.2673	0.267	0.2716
Ω_k	0.0084	0.0084	0.0084	0.0084	0.0084	-0.0133

We see that the ΛCDM model is more sensitive to variations of Ω_m and Ω_k and the better result for this model is achieved. Here optimal values of the model parameters are determined by the constraints (3.4). We impose these constraints on the model [9] though they are not strictly applicable to it. In this model $\min\chi^2$ weakly depends on Ω_0 and Ω_k, so we can not diminish χ^2 appreciably if we slightly broaden the limitations (3.4).

In Fig. 4 one can see evolution of the scale factor $a(\tau)$, (and b for the model [9]), the acceleration parameter $-q(\tau)$ and density $\bar\rho(\tau)$ for the ΛCDM model and the model [9] with $d = 1$ (solid lines), $d = 2$ (dots) and $d = 6$ (dash-dotted lines). For all these models we use the optimal parameters from Table II.

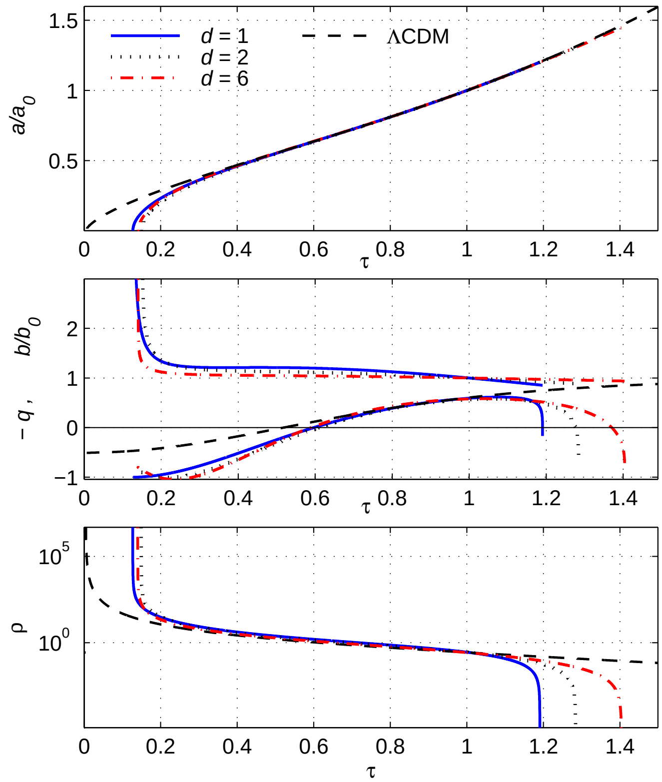

FIG. 4: Scale factors $a(\tau)$, $b(\tau)$, acceleration parameter $-q(\tau)$ and density $\bar\rho(\tau)$ for the optimal solutions from Table II

Evolution of the scale factor $a(\tau)$ for the model [9] with different d and for the ΛCDM model is very close up to $z \simeq 1.5$ ($a > 0.4a_0$), before this epoch the ΛCDM model demonstrates slower expansion. This difference is more visible for the acceleration graphs $-q(\tau)$. The scale factor b for the case [9] diminishes to $b \simeq b_0$ according to the mentioned above compactification scheme (compare with the regular solutions in Fig. 1).

Behavior of cosmological solutions in the future for both models is also different. The ΛCDM model demonstrates unlimited accelerated expansion whereas for the model [9] the acceleration turns into deceleration and inevitability results in the above mentioned zero density $\bar\rho$ at $\tau = \tau_*$ with nonphysical values $\bar\rho < 0$ for $\tau > \tau_*$. The finite lifetime of this universe depends on d, it is the smallest for $d = 1$. In the next section we discuss how to eliminate this essential drawback of the model.

IV. MODIFICATION OF THE MODEL

We have noted that all cosmological solutions in the model [9] have the finite-time future singularity. This inevitable "end of the world" is connected with the chosen power law dependence (2.15) of pressure $\bar p_b$ in extra dimensions on density $\bar\rho$. The terms with the factor $\bar p_b$ in equations (2.14) for $d = 1$ or (2.20) for $d > 1$ determine rate of density decreasing when $\bar\rho$ is small at the

end of its evolution. In this case the leading terms in the mentioned equations are

$$\bar\rho' \simeq \begin{cases} \bar p_b A', & d=1, \\ -d\bar p_b B', & d>1, \end{cases} \qquad \bar\rho\to 0. \tag{4.1}$$

For $\bar\rho \to 0$ we have nonzero values A' and B', so for the weak power law dependence (2.15) the approximate equation (4.1) $\bar\rho' \simeq -C\bar\rho^{1-\gamma}$ has the finite solution

$$\bar\rho \simeq \big[\gamma C(\tau_* - \tau)\big]^{1/\gamma}. \tag{4.2}$$

To avoid this finiteness we are to modify the equation of state (power law dependence) (2.15) of the model [9] for small $\bar\rho$. In particular, a linear dependence for $\bar\rho$ close to zero

$$\bar p_b = w_0\bar\rho, \qquad \bar\rho\to 0 \tag{4.3}$$

ensures infinite evolution with positive density.

The linear law (4.3) for all $\bar\rho$ does not describe the observed accelerated expansion. For good agreement with observations we are to search an equation of state $\bar p_b(\bar\rho)$ with slower growth of $|\bar p_b|$ at high $\bar\rho$ similar to Eq. (2.15). We suggest the appropriate variant of this dependence

$$\bar p_b = \bigg(w_1 + \frac{w}{\rho_0+\bar\rho}\bigg)\bar\rho \tag{4.4}$$

with the linear law (4.3) for $\bar\rho \ll \rho_0$ (here $w_0 = w_1 + w/\rho_0$) and another linear law $\bar p_b \simeq w_1\bar\rho$ for $\bar\rho \gg \rho_0$.

The model (2.9) – (2.12) or (2.20) for $d>1$ with the linear-fractional equation of state (4.4) makes it possible to avoid finite lifetime of the type (4.2) and to transform it into the exponential asymptotic behavior

$$\bar\rho \sim \exp(-C\tau), \qquad C = \mathrm{const}\cdot\bigg(w_1+\frac{w}{\rho_0}\bigg). \tag{4.5}$$

This behavior results from the equation $\bar\rho' \simeq -C\bar\rho$ and may be observed in graphs $\bar\rho(\tau)$ in Fig. 5.

For the model with Eq. (4.4) we can find optimal values of parameters w, w_1, ρ_0, Ω_0, Ω_k presented in Table III and achieve better agreement with the supernovae data [19] than for the models ΛCDM and [9] with Eq. (2.15). Cosmological solutions for the model with Eq. (4.4) and parameters from Table III are shown in Fig. 5.

TABLE III: Optimal parameters for the model with Eq. (4.4), $\rho_0 = 0.005$ is fixed

d	1	2	3	6	10	ΛCDM
$\min\chi^2$	562.898	562.814	562.79	562.766	562.757	563.058
w	-1.603	-1.026	-0.84	-0.658	-0.587	-
w_1	-0.195	-0.343	-0.387	-0.426	-0.439	-
Ω_0	0.2815					0.2716
Ω_k	-0.0133					-0.0133

We see in Table III that the accuracy of the model with Eq. (4.4) increases (χ^2 diminishes) for large d, unlike in the case with Eq. (2.15) in Table II. We should note that the values χ^2 in Table III are not absolutely minimal, because we fixed the parameter $\rho_0 = 0.005$. It is interesting, that for all d we can achieve smaller values $\min\chi^2$, if we take smaller values of ρ_0. But if $\rho_0 \to 0$, the factor C in the exponent (4.5) tends to infinity, the density $\bar\rho$ decreases too rapidly and the picture of vanishing $\bar\rho$ looks like in the finite case in Fig. 4. So we put the restriction $\rho_0 \ge 0.005$ to exclude this almost instantaneous transition to the state with $\bar\rho \simeq 0$. Under this constraint we have the optimal value $\rho_0 = 0.005$ and also $\Omega_0 = 0.2815$, $\Omega_k = -0.0133$ for all d.

Fig. 5 demonstrates cosmological solutions for the model with Eq. (4.4) with the optimal values of parameters from Table III. For both models Eqs. (4.4) and (2.15) in Figs. 4 and 5 the acceleration epoch is finite and its duration depends on d in the same manner. But after this epoch for the model with Eq. (4.4) we see here infinite decelerated expansion.

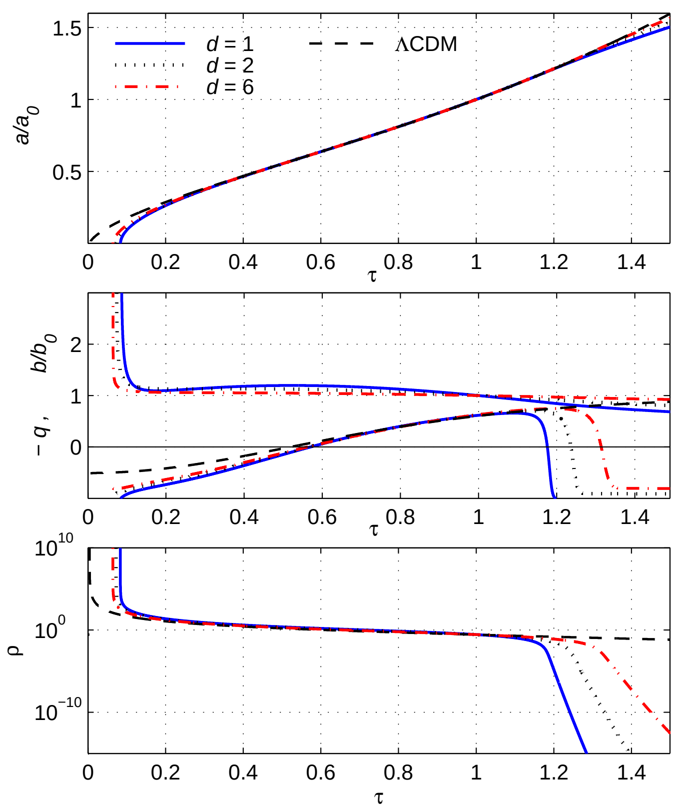

FIG. 5: Scale factors $a(\tau)$, $b(\tau)$, functions $-q(\tau)$ and $\bar\rho(\tau)$ for the model with Eq. (4.4) with the parameters from Table III

Graphs of $a(\tau)$, $-q(\tau)$ and $\bar\rho(\tau)$ here are more close to the dashed lines for the ΛCDM model during the acceleration epoch than the similar curves in Fig. 4. But after the mentioned epoch predictions of the ΛCDM model and the model (4.4) sharply diverge.

In Fig. 6 we present how the model with Eq. (4.4) for

$d = 1, 2, 6$ and the ΛCDM model describe the supernovae data from the site [19] (dots) in the z-D_L plane. All these models were considered below in Fig. 5, we use the same optimal parameters from Table III and the same notations for the curves, in particular, the dashed line for the ΛCDM model.

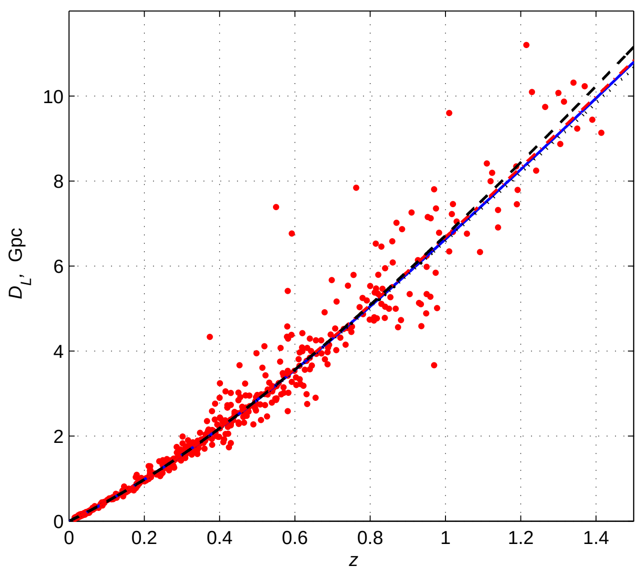

FIG. 6: Luminosity distance D_L in Gpc depending on redshift z for the models ΛCDM and (4.4) with parameters from Table III. Dots are the data [19]

The presented curves are very close in the region $z < 1$, for larger z the ΛCDM line slightly diverges from others. These lines are result of optimal fitting to the observational data [19] (580 dots in Fig. 6). The values χ^2 in Table III show rather good results for the model [9] with Eq. (4.4) for pressure, but these values are not the best fit, because we fixed ρ_0 to avoid the mentioned above sharp transition to $\bar{\rho} \simeq 0$.

V. CONCLUSION

The gravitational model [9] with additional spatial dimensions and anisotropic pressure provides accelerated expansion of the Universe corresponding to observational data for Type Ia supernovae [19]. The authors of Ref. [9] did not consider the case with $d = 1$ additional dimension, but we found that for the chosen in [9] power law equation of state (2.15) it is the case $d = 1$ that yields the best fit (see Tables I and II).

Unfortunately, the model [9] with the power law dependence (2.15) inevitably predicts predicts the finite-time future singularity of Type IV in classification from Refs. [7, 16]. It is connected with vanishing density $\bar{\rho}$ at finite time $\tau = \tau_*$ (and negative density for $\tau > \tau_*$). Evolution of this universe is broken at τ_*, the finite lifetime is shorter for small d (see Fig. 4).

We demonstrated in Sect. IV, that this drawback has technical nature. It is connected with too weak dependence $\bar{p}_b(\bar{\rho})$ for small $\bar{\rho}$ in the power law equation of state (2.15). If we modify this law and choose a linear dependence (4.3) for small $\bar{\rho}$, we obtain an infinite cosmological evolution with positive density (but vanishing at $\tau \to \infty$). We suggest the linear-fractional variant (4.4) of dependence $\bar{p}_b(\bar{\rho})$ to solve the following two problems: (a) to avoid "the end of the world" of the type (4.2) and (b) to describe 580 Type Ia supernovae data points from the site [19]. The dependence (4.4) is a bit more complicated than Eq. (2.15), but it successfully conserves positive density $\bar{\rho}$ during infinite lifetime and fits the data [19] better than the ΛCDM model and the model [9] with Eq. (2.15).

Note that for the model with Eq. (4.4) we practically used only two parameters w and w_1 for fitting. The value ρ_0 was fixed because for very small ρ_0 we have better fit, but the sharp downfall of $\bar{\rho}(\tau)$ to $\bar{\rho} \simeq 0$ looks similar to the mentioned "end of the world". The parameters Ω_0 and Ω_k influence on minimum of χ^2 rather weakly for the model with Eq. (4.4). If we fix, for example, $\Omega_0 = 0.27$ and $\Omega_k = 0$, we obtain minimums for χ^2 differing from results in Table III less then 0.01 for all d.

Cosmological solutions in the model with Eq. (4.4) are divided into regular and singular ones similarly to solutions with Eq. (2.15) shown in Fig. 1. However, Fig. 5 demonstrates that for the optimal values of parameters from Table III solutions with Eq. (4.4) are regular.

It is interesting that the model [9] with both considered variants of dependence $\bar{p}_b$ on $\bar{\rho}$ (2.15) and (4.4) predicts finiteness of the acceleration epoch. Its duration depends on d in the same manner (compare Figs. 4 and 5) and then acceleration sharply turns to deceleration. In the case (2.15) this evolution is broken at $\tau = \tau_*$ with $\bar{\rho}(\tau_*) = 0$, but for the model with Eq. (4.4) the decelerated expansion is infinite and density $\bar{\rho}(\tau)$ tends to zero in the exponential form (4.5).

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